May 2005
revised version
hep-th/0504191

Summing up Non-anti-commutative Kähler Potential [1]

Tomoya Hatanaka [2], Sergei V. Ketov [3], and Shin Sasaki [4]

Department of Physics
Tokyo Metropolitan University
1–1 Minami-osawa, Hachioji-shi
Tokyo 192–0397, Japan

Abstract

We offer a simple non-perturbative formula for the component action of a generic N=1/2 supersymmetric chiral model in terms of an arbitrary number of chiral superfields in four dimensions, which is obtained by the Non-Anti-Commutative (NAC) deformation of a generic four-dimensional N=1 supersymmetric Non-Linear Sigma-Model (NLSM) described by arbitrary Kähler superpotential and scalar superpotential. The auxiliary integrations responsible for fuzziness are eliminated in the case of a single chiral superfield. The scalar potential in components is derived by eliminating the auxiliary fields. The NAC-deformation of the CP^1 Kähler NLSM with an arbitrary scalar superpotential is calculated as an example.

[1] Supported in part by the Japanese Society for Promotion of Science (JSPS)
[2] Email address: thata@kiso.phys.metro-u.ac.jp
[3] Email address: ketov@phys.metro-u.ac.jp
[4] Email address: shin-s@phys.metro-u.ac.jp

1 Introduction

There was a lot of recent activity in investigating various aspects of Non-Anti-Commutative (NAC) superspace and related deformations of supersymmetric field theories (see, e.g., the most recent references [1, 2, 3, 4, 5, 6, 7] directly related to our title, and the references therein for the earlier work in the NAC-deformed N=1 superspace). It is supposed to enhance our understanding of the role of spacetime in supersymmetry, while keeping globally supersymmetric field theory under control.

We work in four-dimensional Euclidean [5] N=1 superspace $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$, and use the standard notation [8]. The NAC deformation is given by

$$\{\theta^\alpha, \theta^\beta\}_* = C^{\alpha\beta} \quad , \tag{1.1}$$

where $C^{\alpha\beta}$ are some constants. The remaining superspace coordinates in the chiral basis ($y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$, $\mu, \nu = 1, 2, 3, 4$ and $\alpha, \beta, \ldots = 1, 2$) still (anti)commute,

$$[y^\mu, y^\nu] = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = [y^\mu, \theta^\alpha] = [y^\mu, \bar{\theta}^{\dot{\alpha}}] = 0 \ . \tag{1.2}$$

The $C^{\alpha\beta} \neq 0$ explicitly break the four-dimensional 'Lorentz' invariance at the fundamental level. The NAC nature of θ's can be fully taken into account by using the Moyal-Weyl-type (star) product of superfields [1] ,

$$f(\theta) * g(\theta) = f(\theta) \ \exp\left(-\frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\vec{\partial}}{\partial\theta^\beta} \right) g(\theta) \ , \tag{1.3}$$

which respects the N=1 superspace chirality. The star product (1.3) is polynomial in the deformation parameter ,

$$f(\theta) * g(\theta) = fg + (-1)^{\deg f} \frac{C^{\alpha\beta}}{2} \frac{\partial f}{\partial\theta^\alpha} \frac{\partial g}{\partial\theta^\beta} - \det C \frac{\partial^2 f}{\partial\theta^2} \frac{\partial^2 g}{\partial\theta^2} \quad , \tag{1.4}$$

where we have used the identity

$$\det C = \tfrac{1}{2}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}C^{\alpha\beta}C^{\gamma\delta} \ , \tag{1.5}$$

and the notation

$$\frac{\partial^2}{\partial\theta^2} = \tfrac{1}{4}\varepsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta} \quad . \tag{1.6}$$

We also use the following book-keeping notation for 2-component spinors:

$$\theta\chi = \theta^\alpha\chi_\alpha \ , \quad \bar{\theta}\bar{\chi} = \bar{\theta}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} \ , \quad \theta^2 = \theta^\alpha\theta_\alpha \ , \quad \bar{\theta}^2 = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}. \tag{1.7}$$

[5] The use of Atiyah-Ward spacetime of the signature $(+, +, -, -)$ is another possibility [9].

The spinorial indices are raised and lowered by the use of two-dimensional Levi-Civita symbols [8]. Grassmann integration amounts to Grassmann differentiation. The anti-chiral covariant derivative in the chiral superspace basis is $\bar{D}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}}$. The field component expansion of a chiral superfield Φ reads

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\chi(y) + \theta^2 M(y) \quad . \tag{1.8}$$

An anti-chiral superfield $\overline{\Phi}$ in the chiral basis is given by

$$\begin{aligned}\overline{\Phi}(y^\mu - 2i\theta\sigma^\mu\bar{\theta},\bar{\theta}) = \;& \bar{\phi}(y) + \sqrt{2}\bar{\theta}\bar{\chi}(y) + \bar{\theta}^2\bar{M}(y) \\ & + \sqrt{2}\theta\left(i\sigma^\mu\partial_\mu\bar{\chi}(y)\bar{\theta}^2 - i\sqrt{2}\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}(y)\right) + \theta^2\bar{\theta}^2\Box\bar{\phi}(y) \ ,\end{aligned} \tag{1.9}$$

where $\Box = \partial_\mu\partial^\mu$. The bars over fields serve to distinguish between the 'left' and 'right' components that are truly independent in Euclidean spacetime.

Our major concern in this Letter is a derivation of the NAC deformation of a generic four-dimensional N=1 supersymmetric action

$$S = \int d^4x \left[\int d^2\theta d^2\bar{\theta}\, K(\Phi^i,\overline{\Phi}^{\bar{j}}) + \int d^2\theta\, W(\Phi^i) + \int d^2\bar{\theta}\,\overline{W}(\overline{\Phi}^{\bar{j}})\right] \tag{1.10}$$

specified by a Kähler superpotential $K(\Phi,\overline{\Phi})$ and a scalar superpotential $W(\Phi)$, in terms of an arbitrary number n of chiral and anti-chiral superfields, $i,\bar{j} = 1,2,\ldots,n$. This problem in four dimensions was addressed in refs. [4, 7], where the perturbative answers (in terms of infinite sums) were found. A similar problem in two dimensions was solved perturbatively in ref. [3], while the non-perturbative summation (in terms of finite functions) was done in ref. [6]. In this Letter we give simple non-perturbative formulas in four dimensions and offer a simple way of their derivation. Our results presumably amount to a full summation of the inifinite sums in refs. [4, 7]. Summing up is crucial for further non-perturbative physical applications of the NAC-deformation and its geometrical interpretation. We also made progress in eliminating the auxiliary integrations and solving the auxiliary field equations, as well as in investigating some concrete examples (see below).

We use the chiral basis, which is most suitable for investigating NAC-deformation, and reduce the most non-trivial problem of calculation of the NAC-deformed Kähler superpotential to that for the NAC-deformed scalar superpotential. The remarkably simple non-trivial results about the NAC-deformed scalar superpotential are already available in refs. [5, 6]. In sect. 2 we describe our idea in the undeformed case (it is not really new there). In sect. 3 we present the results of our calculation for the most general NAC-deformed action (1.10). In sect. 4 we specialize our results to the case of a single chiral superfield and the CP^1 (Kähler) superpotential, as the simplest non-trivial examples. Sect. 5 is our conclusion.

2 Chiral reduction of Kähler superpotential

Let's use the identity

$$\int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi,\overline{\Phi}) = -\tfrac{1}{4}\int d^4y d^2\theta\; \bar{D}^2 K(\Phi,\overline{\Phi})\big| \equiv \int d^4y\, L \quad , \tag{2.1}$$

where $|$ denotes the $\bar{\theta}$-independent part of a superfield, and the constraint $\bar{D}_{\dot{\alpha}}\Phi^i = 0$. Having performed Grassmann differentiation in $\bar{D}^2K$, we arrive at the spacetime (NAC-undeformed) component Lagrangian in the chiral form,

$$L = \int d^2\theta\, V_{(K)}(\Phi^I)\ , \tag{2.2}$$

whose effective scalar superpotential $V_{(K)}(\Phi^I)$ is given by

$$V_{(K)} = -\tfrac{1}{2}K_{,\bar{p}\bar{q}}(\Phi(y,\theta),\bar{\phi}(y))\Phi^{\bar{p}\bar{q}}_{n+1} + K_{,\bar{p}}(\Phi(y,\theta),\bar{\phi}(y))\Phi^{\bar{p}}_{n+2} \tag{2.3}$$

in terms of the *extended* set of the chiral superfields $\Phi^I = \{\Phi^i, \Phi^{\bar{p}\bar{q}}_{n+1}, \Phi^{\bar{p}}_{n+2}\}$. We use the notation (valid for any function $F(\phi,\bar{\phi})$)

$$F_{,i_1 i_2\cdots i_s\bar{p}_1\bar{p}_2\cdots\bar{p}_t} = \frac{\partial^{s+t}F}{\partial\phi^{i_1}\partial\phi^{i_2}\cdots\phi^{i_s}\partial\bar{\phi}^{\bar{p}_1}\partial\bar{\phi}^{\bar{p}_2}\cdots\partial\bar{\phi}^{\bar{p}_t}} \quad , \tag{2.4}$$

and the fact that $\bar{\Phi}^{\bar{j}}\big| = \bar{\phi}^{\bar{j}}(y)$. The additional (composite) $n(n+1)/2$ chiral superfields $\Phi^{\bar{p}\bar{q}}_{n+1}$ and n chiral superfields $\Phi^{\bar{p}}_{n+2}$ are given by

$$\Phi^{\bar{p}\bar{q}}_{n+1}(y,\theta) = \tfrac{1}{2}\,(\bar{D}_{\dot{\alpha}}\bar{\Phi}^{\bar{p}})(\bar{D}^{\dot{\alpha}}\bar{\Phi}^{\bar{q}})\Big| = \bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}} - 2\sqrt{2}i\left(\theta\sigma^\mu\bar{\chi}^{(\bar{p}}\right)\partial_\mu\bar{\phi}^{\bar{q})} - 2\theta^2\partial^\mu\bar{\phi}^{(\bar{p}}\partial_\mu\bar{\phi}^{\bar{q})} \tag{2.5a}$$

and

$$\Phi^{\bar{p}}_{n+2}(y,\theta) = -\tfrac{1}{4}\,\bar{D}^2\bar{\Phi}^{\bar{p}}\big| = \bar{M}^{\bar{p}} + \sqrt{2}i(\theta\sigma^\mu\partial_\mu\bar{\chi}^{\bar{p}}) + \theta^2\Box\bar{\phi}^{\bar{p}}\ . \tag{2.5b}$$

Therefore, the NAC deformation of the Kähler superpotential is not an independent problem, since it is derivable from the NAC deformation of the effective scalar superpotential (2.3) having the extended number of chiral superfields.

The simple non-perturbative form of an arbitrary NAC-deformed scalar superpotential V, depending upon a single chiral superfield Φ of eq. (1.8), was first calculated in ref. [5],

$$\int d^2\theta\, V_*(\Phi) = \tfrac{1}{2c}\left[V(\phi+cM) - V(\phi-cM)\right] - \frac{\chi^2}{4cM}\left[V_{,\phi}(\phi+cM) - V_{,\phi}(\phi-cM)\right]\ , \tag{2.6}$$

where we have introduced the (finite) deformation parameter

$$c = \sqrt{-\det C}\ . \tag{2.7}$$

As is clear from eq. (2.6), the NAC-deformation in the single superfield case gives rise to the scalar potential split controlled by the auxiliary field M. When using an identity

$$f(x+a) - f(x-a) = a\frac{\partial}{\partial x}\int_{-1}^{+1} d\xi\, f(x+\xi a) \ , \tag{2.8}$$

valid for any function f, we can rewrite eq. (2.6) to the equivalent form,

$$\int d^2\theta\, V_*(\Phi) = \tfrac{1}{2}M\frac{\partial}{\partial\phi}\int_{-1}^{+1} d\xi\, V(\phi+\xi cM) - \frac{1}{4}\chi^2\frac{\partial^2}{\partial\phi^2}\int_{-1}^{+1} d\xi\, V(\phi+\xi cM) \ , \quad (2.9a)$$

which is very suitable for an immediate generalization to the case of several chiral superfields (*cf.* ref. [6]),

$$\int d^2\theta\, V_*(\Phi^I) = \tfrac{1}{2}M^I\frac{\partial}{\partial\phi^I}\widetilde{V}(\phi,M) - \frac{1}{4}(\chi^I\chi^J)\frac{\partial^2}{\partial\phi^I\partial\phi^J}\widetilde{V}(\phi,M) \ , \quad (2.9b)$$

where the auxiliary pre-potential $\widetilde{V}$ has been introduced [6],

$$\widetilde{V}(\phi,M) = \int_{-1}^{+1} d\xi\, V(\phi^I+\xi cM^I) \ . \tag{2.10}$$

Therefore, the NAC-deformation of a generic scalar superpotential V results in its smearing or fuzziness controlled by the auxiliary fields M^I [6].

3 The NAC-deformed Kähler superpotential

Our general result in components is just given by eq. (2.9b) after a subsitution of eq. (2.3) in terms of the definitions of sect. 2. We find (*cf.* refs. [3, 4, 6, 7])

$$\begin{aligned} L \;=\;& \tfrac{1}{2}M^iY_{,i} + \tfrac{1}{2}\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}Z_{,\bar{p}\bar{q}} + \tfrac{1}{2}\Box\bar{\phi}^{\bar{p}}Z_{,\bar{p}} - \tfrac{1}{4}(\chi^i\chi^j)Y_{,ij} \\ & - \tfrac{1}{2}i(\chi^i\sigma^\mu\bar{\chi}^{\bar{p}})\partial_\mu\bar{\phi}^{\bar{q}}Z_{,i\bar{p}\bar{q}} - \tfrac{1}{2}i(\chi^i\sigma^\mu\partial_\mu\bar{\chi}^{\bar{p}})Z_{,i\bar{p}} \ , \end{aligned} \tag{3.1}$$

where we have introduced the (component) smeared Kähler pre-potential

$$Z(\phi,\bar{\phi},M) = \int_{-1}^{+1} d\xi\, K^\xi \quad \text{with} \quad K^\xi \equiv K(\phi^i+\xi cM^i,\bar{\phi}^{\bar{j}}) \ , \quad (3.2a)$$

as well as the extra (auxiliary) pre-potential

$$Y(\phi,\bar{\phi},M,\bar{M}) = \bar{M}^{\bar{p}}Z_{,\bar{p}} - \tfrac{1}{2}(\bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}})Z_{,\bar{p}\bar{q}} + c\int_{-1}^{+1} d\xi\xi\left[\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}K^\xi_{,\bar{p}\bar{q}} + \Box\bar{\phi}^{\bar{p}}K^\xi_{,\bar{p}}\right] \ . \quad (3.2b)$$

We verified that eq. (3.1) reduces to the standard (Kähler) N=1 supersymmetric non-linear sigma-model in the limit $c\to 0$. Also, in the case of a free (bilinear) Kähler potential $K=\delta_{i\bar{j}}\Phi^i\bar{\Phi}^{\bar{j}}$, there is no deformation at all.

Given, in addition, an independent scalar superpotential $W(\Phi)$, as in eq. (1.10), the following component terms are to be added to eq. (3.1):

$$L_{\text{potential}} = \tfrac{1}{2}M^i\widetilde{W}_{,i} - \tfrac{1}{4}(\chi^i\chi^j)\widetilde{W}_{,ij} + \bar{M}^{\bar{p}}\bar{W}_{,\bar{p}} - \tfrac{1}{2}(\bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}})\bar{W}_{,\bar{p}\bar{q}} \ , \tag{3.3}$$

where we have used the fact that the anti-chiral scalar superpotential terms are inert under the NAC-deformation, and we have introduced the smeared scalar pre-potential [6]

$$\widetilde{W}(\phi, M) = \int_{-1}^{+1} d\xi\, W(\phi^i + \xi c M^i) \ . \tag{3.4}$$

Though our general results of this section are quite explicit and non-perturbative, as regards their applications, it is desirable to perform all integrations over ξ (thus evaluating the smearing effects) and eliminate the auxiliary fields $(M,\bar{M})$ by using their algebraic equations of motion. It is clear that the ξ-integrals can be evaluated once the Kähler and scalar functions, K and W, are specified. The anti-chiral auxiliary fields $\bar{M}^{\bar{p}}$ enter the action linearly (as Lagrange multipliers), so that their algebraic equations of motion determine the auxiliary fields $M^i = M^i(\phi,\bar{\phi})$,

$$\tfrac{1}{2}M^i Z_{,i\bar{p}} + \bar{W}_{,\bar{p}} = 0 \ . \tag{3.5}$$

The bosonic scalar potential in components is thus given by

$$V_{\text{scalar}}(\phi,\bar{\phi}) = \tfrac{1}{2}M^i\widetilde{W}_{,i}\Big|_{M=M(\phi,\bar{\phi})} \ , \tag{3.6}$$

in agreement with ref. [6]. In the next sect. 4 we study some examples, by restricting our general results to the case of a single chiral superfield, and then to the case of the CP^1 Kähler potential.

4 Examples

It is quite natural to begin with an example of a *single* chiral superfield, while keeping arbitrary both Kähler and scalar functions. In this case all the ξ-integrations can be easily performed, e.g., by using eq. (2.8) and the related identity obtained by differentiating eq. (2.8) with respect to the parameter a,

$$f(x+a) + f(x-a) = \int_{-1}^{+1} d\xi\, f(x+\xi a) + a\frac{\partial}{\partial x}\int_{-1}^{+1} d\xi\, \xi f(x+\xi a) \ . \tag{4.1}$$

Of course, one would get the same results by using the chiral reduction of sect. 2 and the crucial identity (2.6) from the very beginning. We found some cancellations

amongst the bosonic terms, with the result

$$
\begin{aligned}
L_{\text{bosonic}} = &+ \tfrac{1}{2}\partial^{\mu}\bar{\phi}\partial_{\mu}\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \tfrac{1}{2}\Box\bar{\phi}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{\bar{M}}{2c}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{2c}\left[W(\phi+cM)-W(\phi-cM)\right]+\bar{M}\frac{\partial\bar{W}}{\partial\bar{\phi}} \quad .
\end{aligned}
\tag{4.2}
$$

The bosonic terms are to be supplemented by the following fermionic terms:

$$
\begin{aligned}
L_{\text{fermionic}} = &- \frac{1}{4c}\bar{\chi}^2\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{i}{2cM}(\chi\sigma^{\mu}\bar{\chi})\partial_{\mu}\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{i}{2cM}(\chi\sigma^{\mu}\partial_{\mu}\bar{\chi})\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{\bar{M}}{4cM}\chi^2\left[K_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4M}\chi^2\partial^{\mu}\bar{\phi}\partial_{\mu}\bar{\phi}\left[K_{,\phi\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\phi\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{4cM^2}\chi^2\partial^{\mu}\bar{\phi}\partial_{\mu}\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4M}\chi^2\Box\bar{\phi}\left[K_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{4cM^2}\chi^2\Box\bar{\phi}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{8cM}\chi^2\bar{\chi}^2\left[K_{,\phi\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\phi\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4cM}\chi^2\left[W_{,\phi}(\phi+cM)-W_{,\phi}(\phi-cM)\right]-\tfrac{1}{2}\bar{\chi}^2\bar{W}_{,\bar{\phi}\bar{\phi}} \ .
\end{aligned}
\tag{4.3}
$$

In the case of the CP^1 Kähler potential

$$
K(\phi,\bar{\phi}) = \ln(1+\phi\bar{\phi}) \quad , \tag{4.4}
$$

the auxiliary field equation (3.5) gives rise to a quadratic equation on M, whose roots are given by

$$
M = \frac{1\pm\sqrt{1+(2c\bar{\phi}(1+\phi\bar{\phi})\bar{W}')^2}}{2c^2\bar{\phi}^2\bar{W}'} \quad , \tag{4.5}
$$

where we have used the notation $\bar{W}'=\partial\bar{W}/\partial\bar{\phi}$. Taking the anticommutative limit $c\to 0$ implies that we should choose minus in eq. (4.5). The scalar potential is given by eq. (3.6), after substituting eqs. (3.4) and (4.5) overthere.

More examples and applications will be considered elsewhere [10].

5 Conclusion

A comparison to the NAC-deformed N=2 NLSM in *two* dimensions [3, 6] is possible after dimensional reduction of our results in sects. 3 and 4 by assuming $\partial_3 = \partial_4 = 0$. As was already demonstrated in ref. [6], the infinite series found in ref. [3] can be resummed in terms of the 'minimally' deformed Kähler potential and superpotential in the sense of eqs. (3.2a) and (3.4), plus some additional 'non-minimal' terms with deformed coupling as in eq. (3.2b), in *precise* agreement with our basic formulae (2.6) and (2.9). [6] As regards the infinite series found in refs. [4, 7] in four dimensions, those results seem to be very complicated to allow us a direct comparison. An explicit resummation is necessary not only (i) for comparison but also (ii) checking the locality of the deformed action in spacetime, (iii) verifying the auxiliary fields to be still non-propagating, and (iv) ultimately solving the auxiliary field equations. We believe that our results in this Letter are useful for all those purposes, because we addressed the most general case, formulated our results in a compact and transparent form, and offered a clear 'short cut' for their easy derivation.

As is well known in the theory of NLSM (see e.g., ref. [11]), the so-called quotient construction (or gauging isometries of the NLSM target space) can be used to represent some NLSM with homogeneous target spaces as the gauge theories. It was used in ref. [12] to construct the NAC-deformed supersymmetric NLSM in four dimensions with the CP^n target space, by combining the quotient construction with the results of ref. [1] about the NAC-deformed supersymmetric gauge theories. As is clear from our results about generic NAC-deformed NLSM in sect. 3, the NAC deformation of a Kähler potential is controlled by the auxiliary fields entering the deformed Kähler potential in the highly non-linear way. The auxiliary fields are determined by their algebraic equations of motion that are also dependent upon a superpotential, even when all fermions are ignored. As a result, the NAC deformation of Kähler geometry is controlled by a scalar superpotential! It is in drastic contrast with the standard supersymmetric (Kähler) NLSM in undeformed superspace, whose target space geometry is unaffected by a scalar superpotential. In the absence of a scalar superpotential, $W = \bar{W} = 0$, we found that the NAC deformation $c \neq 0$ does *not* affect the supersymmetric CP^1 NLSM action at all, in agreement with ref. [12].

A detailed analysis of the actions (3.1), (4.2) and (4.3), including the deformed supersymmetric CP^n NLSM, will be given elsewhere [10].

[6] The split (2.6) of a NAC-deformed superpotential in the case of a single chiral superfield was found in ref. [5]. The smearing (2.9) and (2.10) of NAC-deformed Kähler potential and superpotential in the case of several chiral superfields was found in ref. [6].

Acknowledgements

The authors are grateful to Y. Kobayashi for discussions, B. Chandrasekhar, A. Kumar and A. Lerda for correspondence, and the referee for his careful reading of our manuscript and useful suggestions.

References

[1] N. Seiberg, JHEP **0306** (2003) 010 [hep-th/0305248]

[2] M. Billo, M. Frau, F. Lonegro and A. Lerda, *N=1/2 quiver gauge theories from open strings with R-R flixes*, hep-th/0502084

[3] B. Chandrasekhar and A. Kumar, JHEP **0403** (2004) 013 [hep-th/0310137];
B. Chandrasekhar, Phys.Rev. **D70** (2004) 125003 [hep-th/0408184];
B. Chandrasekhar, *N=2 σ-model action on non(anti)commutative superspace*, hep-th/0503116

[4] O.D. Azorkina, A.T. Banin, I.L. Buchbinder and N.G. Pletnev, *Generic chiral superfield model on nonanticommutative $N = \frac{1}{2}$ superspace*, hep-th/0502008

[5] T. Hatanaka, S. V. Ketov, Y. Kobayashi and S. Sasaki, Nucl. Phys. **B716** (2005) 88 [hep-th/0502026]

[6] L. Alvarez-Gaume and M. Vazquez-Mozo, *On nonanticommutative N=2 sigma-models in two dimensions*, hep-th/0503016

[7] T. A. Ryttov and F. Sannino, *Chiral models in nonanticommutative N=1/2 four-dimensional superspace*, hep-th/0504104

[8] J. Wess and J. Bagger, *Supersymmetry and Supergravity* , Princeton University Press, 1992

[9] S. J. Gates, Jr., S. V. Ketov and H. Nishino, Nucl. Phys. **B393** (1993) 149 [hep-th/9207042]

[10] T. Hatanaka, S. V. Ketov, Y. Kobayashi and S. Sasaki, in preparation

[11] S. V. Ketov, *Quantum Non-linear Sigma-Models*, Springer-Verlag, Heidelberg, 2000

[12] T. Inami and H. Nakajima, Progr. Theor. Phys. **111** (2004) 961 [hep-th/0402137].